A power-based perspective in modeling and control of switched power converters [Past and Present]

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Published in:
IEEE Industrial Electronics Magazine

DOI:
10.1109/MIE.2007.357170

Publication date:
2007

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A Power-Based Perspective in Modeling and Control of Switched Power Converters

Switched power converters today are essential for high-performance energy control in a large variety of applications. This varies from simple dc-dc, ac-dc, dc-ac, and ac-ac converters for use in commercial electrical equipment, to high-tech converters in space and noncivilian applications. A central concern in the design of switched-mode power converters is the improvement of their dynamic behavior. In the last 40 years, this area has seen major developments from design and analysis perspectives. The analysis, however, is mainly based on small-signal analysis (linearization) and the controller design is usually related to linear proportional-integral differential (PID) control techniques. Due to increasing demands on efficient energy management and conversion, together with demands on reduced harmonic generation, higher bandwidths, and reliability, classic (linear) analysis and controller design tools have reached their limits. For that reason, the development of dedicated tools that take the systems nonlinearities into account is of utmost importance.

Passivity-Based Control

In the last decade, switched power converters have been considered from an energy-based modeling and control perspective. By using the Euler-Lagrange and Hamiltonian system descriptions from classical mechanics, important physical properties such as energy and interconnection are underscored. In combination with passivity, a characteristic of many physical systems, these physical properties can advantageously be exploited at the feedback controller design stage, often resulting in nonlinear controllers. In particular, nonlinear passivity-based control (PBC) algorithms for switched converters have proven to be an interesting alternative to other, mostly linear, control techniques [1]. The terminology PBC has its origins in the field of robotics and directly exploits the energy and dissipation structure of the Euler-Lagrange or Hamiltonian models for the systems. The PBC objective is usually achieved through an energy-reshaping process and damping injection to modify the dissipation structure of the system. From a circuit-theoretic perspective, a PBC forces the closed-loop dynamics to behave as if there are artificial resistors, i.e., the control parameters, added to the system. However, the key question that remains is where to inject the damping and how to tune the controller.

To develop some guidelines for tuning the control parameters such that the closed loop system is able to suppress the influence of load variations, $\mathcal{L}_2$-gain analysis techniques can be used (see the references in [2]). Unfortunately, the necessary calculations become quite complex, especially when dealing with large converter structures. This motivates why the injected damping in the Euler-Lagrange and Hamiltonian based controller designs is often located based on the form of the open-loop dissipation structure, i.e., damping is added to those states that do not contain any damping terms a priori. However, such a PBC scheme might lead to a regulator that is highly sensitive to load variations.

In the early 1960s, J.K. Moser developed a mathematical analysis to study the stability of electrical networks containing tunnel diodes. His method is based on the introduction of a scalar function called the mixed-potential function. The gradient of the mixed-potential function leads to the equations of motion for the nonlinear network. Four years later, Moser generalized the method together with R.K. Brayton in [3]. The process of constructing the mixed-potential involves the definition of the current- and voltage potentials (content and co-content) associated with the resistors, sources, and transformers in the network and the expression of these functions in terms of the inductor currents and capacitor voltages. This in contrast to related methods like the classical Euler-Lagrange or Hamiltonian formulations that involve the energy storage in the inductors and capacitors in terms of charges and fluxes.

The principal application of the concept of the mixed-potential function concerns its use in determining stability criteria for nonlinear networks. In this approach the mixed potential will be used as a starting point to define a power-based Lyapunov function for application of Lyapunov's second method (a stability theory for dynamical systems named after the Russian mathematician A.M. Lyapunov). Such applications are explored in [3], where the authors give examples of their stability theory applied to topologically complete networks. A strong feature of the method is that it can also be applied to networks with negative resistances. During the last four decades, several notable extensions and generalizations of the Brayton-Moser theory have been presented in the literature (see e.g., [2] and the references therein).

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Recently, in [2] the PBC methodology for switched converters is rewritten in terms of the Brayton-Moser (BM) equations. The BM equations, as originally proposed in the early 1960s [3], were primarily used to describe and analyze the dynamics of nonlinear RLC networks using a single scalar function. This function, called the *mixed potential*, has the dimensions of power and can be used to investigate the stability of the network. In contrast to the Euler-Lagrange and Hamiltonian modeling frameworks, a practical advantage of the BM equations is that they are naturally described in terms of easily measurable quantities, i.e., the inductor currents and capacitor voltages, instead of the fluxes and charges. Another advantage of using the BM framework for PBC is that practical guidelines for the structure of the injected damping follow naturally and can be justified mathematically. Furthermore, the controller tuning problem can be addressed using modified versions of Brayton and Moser's stability theorems.

Switched Mixed Potential

In the context of switched power converters, the BM equations first need to be accommodated for the inclusion of switches. For converters containing a single controllable switch, denoted by $\sigma \in \{0, 1\}$, the switched BM equations take the form [2]

$$-L(i)\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{\partial P^{\sigma}(i,u)}{\partial i},$$
$$C(u)\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\partial P^{\sigma}(i,u)}{\partial u}, \qquad (1)$$

where i and u are, respectively, the currents and voltages associated to the (possibly nonlinear) inductors $L(i)$ and capacitors $C(u)$. The scalar function P^{σ} is the switched mixed potential. The switched mixed potential is determined by

$$P^{\sigma}(i,u) = (1-\sigma)P^{0}(i,u) + \sigma P^{1}(i,u), \qquad (2)$$

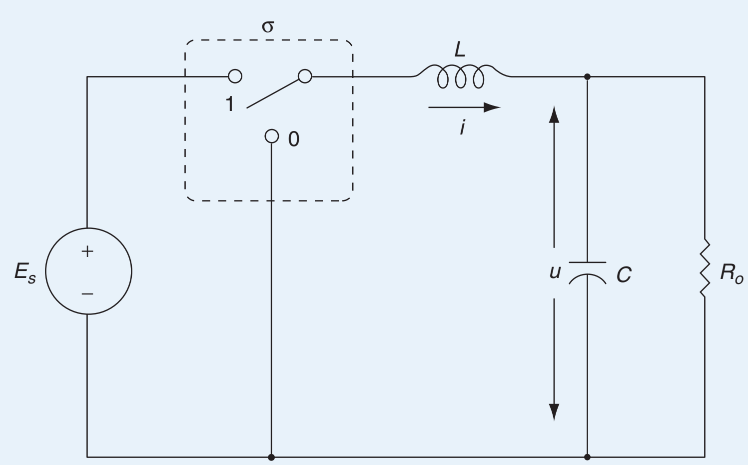
where $P^{0}(i,u)$ and $P^{1}(i,u)$, respectively, represent the mixed potentials for the switch in position 0 (OFF) and 1 (ON). A simple example using the elementary buck (step down) converter is shown in Figure 1. Observe that the equations in (1) correspond to Kirchhoff's voltage and current law, respectively. The inclusion of multiple switches is easily accomplished by appropriately extending (2). Noncontrollable switches, such as diodes, can be treated as nonlinear resistors.

Damping Injection via the BM Framework

Since the PBC methodology requires a pulse-width modulation (PWM) device to determine the switch position, the next step is to consider the *average* BM model. Under the assumption that the PWM switching frequency is sufficiently high, the switched-mode BM description is replaced by its continuous-time averaged approximate BM model. This means that the actual state variables, i and u, in (1) and (2) are replaced by their average versions, say $\langle i\rangle$ and $\langle u\rangle$, and the switch position σ is replaced by its corresponding duty ratio μ. For the buck converter example, the average mixed-potential becomes

$$P^{\mu}(\langle i\rangle, \langle u\rangle) = \langle i\rangle\langle u\rangle - \frac{\langle u\rangle^2}{2R_o} - \mu\langle i\rangle E_s. \qquad (3)$$

Roughly speaking, the next step to obtain a PBC is to make a copy of (3) in terms of some desired auxiliary state variables (the controller state variables) and add additional damping in terms of the error state variables (i.e., the difference between the average converter state variables and the desired auxiliary state variables). (Note that adding additional damping to (3) results in a modification of the mixed potential, thus reshaping the power structure of the converter. See [2] for details.) As discussed before, at this point one is usually tempted to add damping on those states that do not contain any damping a priori. For the buck converter example this would mean that, since the output capacitor voltage already contains a damping term given by the load resistor, damping should be added on the current coordinate. However, straightforward application of the modified BM stability theorems in [2] suggest that there are two natural ways of injecting additional damping: series damping injection (current feedback) and parallel damping injection (voltage feedback). Series damping PBC forces the closed-loop dynamics to act as if there are resistors in series with the inductances, while parallel damping PBC corresponds to "injecting" resistors in parallel with the capacitors. Figure 2 illustrates a circuit-theoretic interpretation of the buck converter in closed-loop with, respectively, a series and a parallel damping



$$\left.\begin{aligned} P^0(i,u) &= iu - \frac{u^2}{2R_o} \\ P^1(i,u) &= P^0(i,u) - iE_s \end{aligned}\right\} \Rightarrow P^{\sigma}(i,u) = iu - \frac{u^2}{2R_o} - \sigma i E_s.$$

FIGURE 1 — The single switch buck converter and its switched mixed potential.

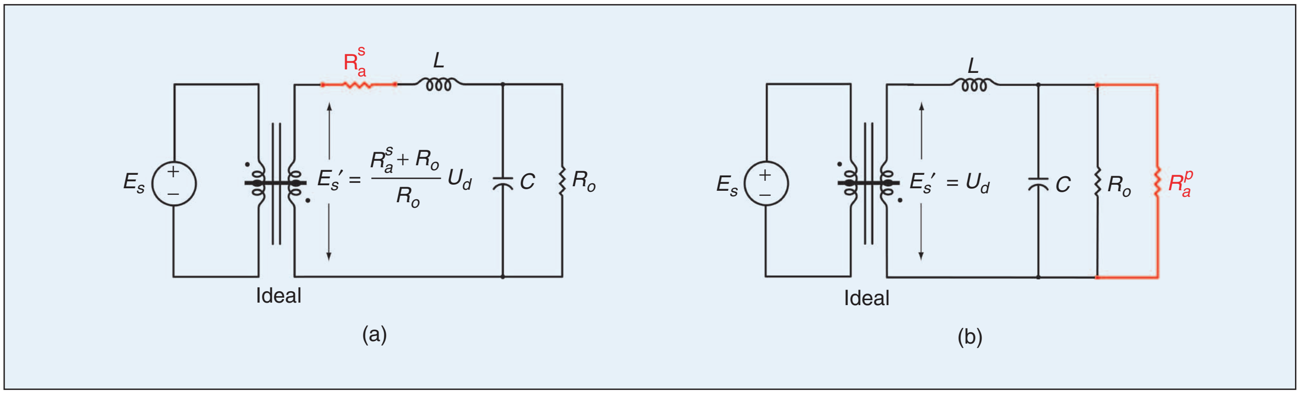


FIGURE 2 — Closed-loop interpretation of a (a) series and (b) parallel damping PBC buck converter. Here R_a^* denotes the injecting damping resistor and U_d denotes the desired output voltage.

PBC. As can be observed, the injection of a series resistance R_a^s introduces the effect of a voltage divider, with a ratio $R_o^{-1}(R_a^s + R_o)$. Hence, since this ratio depends on the load resistor R_o, the closed loop will be highly sensitive to load variations. To overcome this problem, one usually adds an adaptive mechanism to estimate the load resistance, at the cost of computational complexity. On the other hand, the parallel damping PBC strategy does not suffer from this effect. Here the voltage divider ratio equals unity and does not depend on the load.

Besides the type of injected damping, the modified BM stability theorems also provide information on the lower and upper bounds for tuning the control parameters. For the buck converter example, good performance is ensured if either

$$R_a^s > \sqrt{\frac{L}{C}}$$

for series damping injection or

$$R_a^p < \sqrt{\frac{L}{C}}$$

for parallel damping injection. These fairly sharp criteria form a systematic and straightforward tool for solving the tuning problem for a general class of PBC switched power converters. The interested reader is referred to [2] for more details.

Further Developments

A possible drawback of the PBC method in [1] and [2] is that it relies on a (partial) system inversion. This usually results in indirect regulations schemes to control non-minimum phase outputs. Recently, a new control paradigm for nonlinear electrical circuits was presented using the mixed-potential as a *control-Lyapunov* function. On-going research aims at extending the paradigm to switched power converters in order to avoid the system inversion problem.

(continued on page 54)

Past and Present

(continued from page 9)

Biographies

Dimitri Jeltsema received the B.Sc. degree in electrical engineering from the Rotterdam School of Engineering, The Netherlands, and the M.Sc. degree in systems and control engineering from the University of Hertfordshire, United Kingdom, in 1996 and 2000, respectively. In May 2005 he received the Ph.D. degree (cum laude) from Delft University of Technology, The Netherlands. He is a post-doctoral researcher at the same institute. His research interests include nonlinear circuit theory, power electronics, and physical modeling and control techniques. He is a Member of the IEEE.

Jacquelien M.A. Scherpen received her M.Sc. and Ph.D. degrees in applied mathematics from the University of Twente, The Netherlands, in 1990 and 1994, respectively. After a post-doctoral year, she was appointed as assistant/associate professor at the Delft Center for Systems and Control, Delft University of Technology, The Netherlands, until 2006. Currently she is a full professor at the Faculty of Mathematics and Natural Sciences, University of Groningen, The Netherlands. Her research interests include nonlinear model reduction methods, realization theory, nonlinear control methods, in particular modeling and control of physical systems with applications to nonlinear electrical circuits. She is a Senior Member of the IEEE. She was associate editor for *IEEE Transactions on Automatic Control* and is subject editor for the *International Journal of Robust and Nuclear Control.*

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Chapter News

(continued from page 50)

the lab tour was the sensor technologies lab. Research topics of this lab are technologies used in medical applications like the THz multispectral imaging and sensing, trace detection, and ion mobility spectrometry to detect, for instance, explosives worn under the clothes. The last lab of the tour was the medical imaging lab with research topics in magnetic resonance imaging, ultrasound, and molecular imaging. As an example of the research results, GE presented its magnetic resonance spectroscopy and imaging of the prostate as a minimum-invasive method to get information about the health state of this organ.

The joint IAS/PELS/IES German Chapter has elected its new Chapter board for the term 2007/08. The new officers are Prof. Dr. Heinz Van der Broeck (University of Applied Sciences, Köln) as chairman, Dr. Ingo Hahn (Bosch Rexroth) as vice-chairman, Prof. Dr. Axel Mertens (University of Hannover) as secretary, and Dr. Mark Bakran (Siemens) as treasurer.

For further information please visit our Chapter Web site at http://www.ewh.ieee.org/r8/germany/ias-pels.

—Dr. Ingo Hahn
IEEE IAS/PELS/IES, German Chapter

My View

(continued from page 56)

power management. These are just examples, of course.

I have discussed this idea with some of market analyst firms that have expressed good interest in the program. I have also started to interest large companies that are focused in the IES areas or have divisions with a focus in an IES area. The idea has generated surprising interest even in its very early stages.

Our initial subject areas suggest we might consider our first forum to be on the West Coast of the United States to attract the informatics industry in that area. If the concept is successful, we will sponsor a forum in different regions (Americas, Asia, Europe, etc.) with suitable topics for each. Our first forum will be completely separate from other IES activities. With industry interest and time, we might align this to our key conferences like IECON. This would simplify the operation and expose the attendees to consider other events.

The attendees of the Industry Forum are expected to be different than our usual conference and workshop participants, at least for now. I expect both business and technical attendees at the forum. The combination of the market study and our senior speakers should attract chief technology offices and other senior technical members from industry along with some of their business decision counterparts.

Our IES industry Forum does not follow the typical research conferences, such as IECON, but may add a much broader value to IES as a Society that deeply considers the needs and values of business together with the visions and directions of the IES research community. This is certainly our goal, and we hope to achieve it. Success will happen with support from the IEEE community and industry partners; it's not just "another conference."

Biography

Michael W. Condry (condry@ieee.org) received his Ph.D. in computer science from Yale University in 1980. His career included both academic and industry roles at Princeton University, AT&T Bell-Labs, University of Illinois, Sun Microsystems, and Intel. His interests are in computer and networking technologies including Internet applications, security, wireless, and operating systems. He is a Senior Member of the IEEE active in IES AdCom.